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A STEADY STATE ANALYSIS OF THE ADAPTIVE SPECTRAL LINE ENHANCER.(U)
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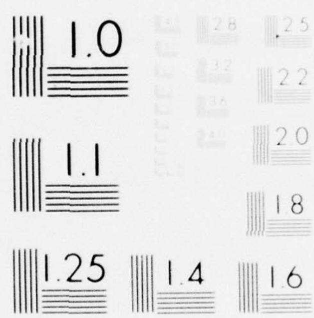
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9 Technical Memo[redacted]

6 A STEADY STATE ANALYSIS OF THE ADAPTIVE SPECTRAL LINE ENHANCER

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ABSTRACT

The adaptive Spectral Line Enhancer (SLE) is a device for automatically prewhitening noise with unknown spectrum and extracting (enhancing) spectral components to which the noise is additive. The steady state performance of the SLE is presented for the general case of multiple spectral lines. The theoretical steady state SNR performance of the SLE is shown to be that of a spectrum analyzer with resolution determined by the length of the adaptively weighted tapped delay line which is used to implement the device.

ADMINISTRATIVE INFORMATION

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I. INTRODUCTION

For many signal detection problems of interest in sonar, a detection processor is required to detect spectral lines in the presence of an additive broadband noise background. The optimum (LMS) detector in this instance requires exact knowledge of both the signal and noise spectra. However, in general this type of a priori knowledge of signal and noise spectra is not available. If the signal is bandlimited (i.e. spectral) and the noise is broadband, then the signal component is correllated for time lags which are large compared to that for the noise. This difference in correlation time can be exploited to build a self-adjusting filter which will automatically extract the spectral component from the broadband noise independent of the frequency of the spectral component. Such an adaptive filter has been called a "Spectral Line Enhancer (SLE) [1]." This document presents a steady state analysis of the SLE, derives the adaptive filter control algorithms and convergence characteristics and considers the limiting case of a white Gaussian noise (WGN) background.

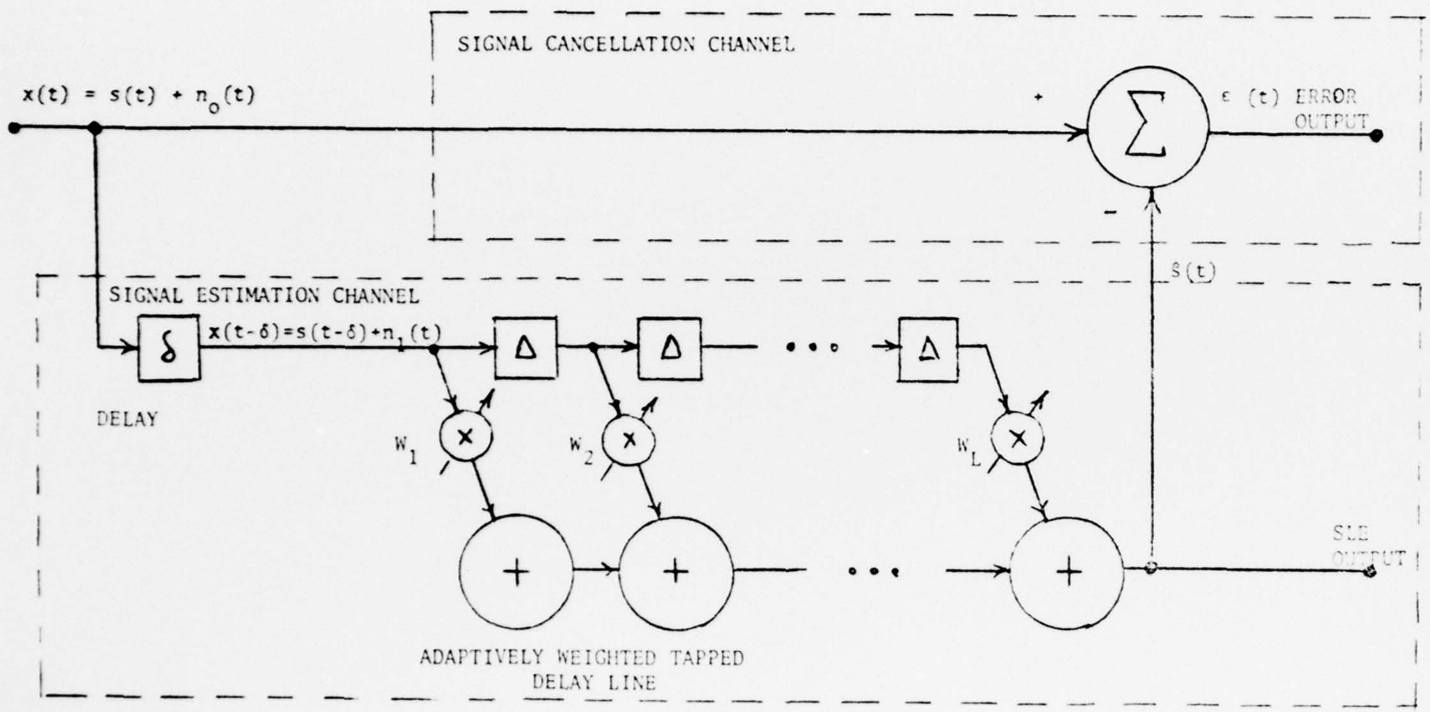
The intent of this document is to show that the SLE is an adaptive noise pre-whitener and signal notch filter which automatically follows any spectral components in the presence of broadband noise. It is observed that, in fact, the local SNR at the output of the notch filter, i.e. within the band defined by the resolution of the adaptive filter, is not increased but rather remains constant. Finally, in general, it is intended to give some additional insight into the SLE operation to assist both in exploiting its potential and understanding its limitations.

II. STEADY STATE ANALYSIS

A functional diagram of the spectral line enhancer (SLE) is given in Figure 1. The signal plus noise input

$$x(t) = s(t) + n(t) \tag{1}$$

is processed in two channels: the signal estimation channel and the signal cancellation channel. The signal estimation channel consists of a bulk delay (δ) which is long enough



NOTE:
$n_1(t) = n_o(t-\delta)$

FIGURE 1. The Spectral Line Enhancer (SLE).

-4-

to de-correlate the broadband additive noise component $n_o(t)$ from $n_o(t-\delta)$, and an adaptively weighted tapped delay line to provide a minimum Mean Squared Error (MSE) estimate, $\hat{s}(t)$ of the signal $s(t)$. The signal cancellation channel subtracts the MSE estimator $\hat{s}(t)$ from the signal-plus-noise input to provide an error signal $\epsilon(t)$. The power in the error signal is then minimized with respect to the weights in the tapped delay line. Since the noise in the signal cancellation channel, $n_o(t)$, is uncorrelated with the noise in the signal estimation channel $n_1(t) = n_o(t-\delta)$, the process of minimizing the error output power works primarily on reducing the signal power in the error output. The MSE estimator, $\hat{s}(t)$, is the desired SLE output. This estimator represents a "cleaned-up", i.e. low additive noise, version of the input signal component $s(t)$ which will be cancelled by subtracting $\hat{s}(t)$.

Let the signal component of the input consist of M spectral terms and be represented by

$$s(t) = \sum_{i=1}^{M} \tilde{s}_i(t) e^{j\omega_i t} \tag{2}$$

where $\tilde{s}_i(t)$ is the complex envelope of a spectral signal at frequency ω_i. If the superscript "T" is the matrix transpose operator, then the signal plus noise vector $\underline{X}$ is defined where

$$\underline{X}^T = \left[x(t-\delta)\, x(t-\delta-\Delta) \cdots x(t-\delta-[L-1]\Delta)\right] \tag{3}$$

$$= \underline{S}^T + \underline{N}^T \tag{4}$$

such that

$$\underline{S}^T = \left[s(t-\delta)\, s(t-\delta-\Delta) \cdots s(t-\delta-[L-1]\Delta)\right] \tag{5}$$

$$\underline{N}^T = \left[n_o(t-\delta)\, n_o(t-\delta-\Delta) \cdots n_o(t-\delta-[L-1]\Delta)\right]. \tag{6}$$

A signal delay matrix

$$D = [\underline{D}_1 \underline{D}_2 \cdots \underline{D}_M] \tag{7}$$

with i^{th} column

$$\underline{D}_i^T = \left[e^{-j\omega_i \delta} \quad e^{-j\omega_i(\delta+\Delta)} \cdots e^{-j\omega_i(\delta+[L-1]\Delta)}\right] \tag{8}$$

in conjunction with a signal complex envelope vector defined as

$$\underline{E} = \left[\tilde{s}_1(t)e^{j\omega_1 t}\ \tilde{s}_2(t)e^{j\omega_2 t}\ \cdots\ \tilde{s}_M(t)e^{j\omega_M t}\right] \tag{9}$$

permits rewriting the transpose of (4) as

$$\underline{X} = D\underline{E} + \underline{N}. \tag{10}$$

Notice that

$$s(t) = \underline{1}^T\underline{E} \tag{11}$$

where $\underline{1}^T = [1\ 1\ \cdots\ 1]$ is an M-element vector. Defining the matrix complex conjugate transpose operator "H", a weight vector $\underline{W}$ is given by

$$\underline{W}^H = [w_1{}^*\ w_2{}^*\ \cdots\ w_L{}^*] \tag{12}$$

where "*" denotes the complex conjugate. The error output is now expressed by

$$\varepsilon(t) = x(t) - \hat{s}(t) \tag{13}$$

$$= x(t) - \underline{W}^H D\ \underline{E} - \underline{W}^H\underline{N} \tag{14}$$

with expected, i.e. steady state power, for uncorrelated signal and noise given by

$$\mathrm{E}\{|\varepsilon(t)|^2\} = \mathrm{E}\{|s(t)|^2\} - 2\mathrm{Re}\left(\underline{W}^H D\mathrm{E}\{\underline{E}\ s^*(t)\}\right) +$$

$$\underline{W}^H D R_{ss} D^H \underline{W} + \mathrm{E}\{|n_o(t)|^2\} + \underline{W}^H R_{NN}\underline{W}. \tag{14a}$$

In (14), the following definitions have been made

$$\mathrm{E}\{\ \cdot\ \} = \text{statistical expectation operator} \tag{15}$$

$$\mathrm{E}\{\underline{E}\underline{E}^H\} = R_{ss}\ \text{(signal correlation matrix)} \tag{16}$$

and

$$\mathrm{E}\{\underline{N}\underline{N}^H\} = R_{NN}\ \text{(noise correlation matrix)}. \tag{17}$$

The minimum value of $E\{|\varepsilon(t)|^2\}$ with respect to $\underline{W}$ is obtained by solving

$$\frac{\partial E\{|\varepsilon(t)|^2\}}{\partial \underline{W}} = -2DR_{ss}\underline{1} + 2[DR_{ss}D^H + R_{NN}]\underline{W} \tag{18}$$
$$= \underline{0}$$

for W_{MSE} to yield (19)

$$W_{MSE} = [DR_{ss}D^H + R_{NN}]^{-1} DR_{ss}\underline{1}. \tag{20}$$

Using the identity

$$[M^H Q^{-1} M + P^{-1}] = P - PM^H[MPM^H + Q]^{-1}MP \tag{21}$$

(20) becomes

$$\underline{W}_{MSE} = R_{NN}^{-1} D(I - [D^H R_{NN}^{-1} D + R_{ss}^{-1}]^{-1} D^H R_{NN}^{-1} D)R_{ss}\underline{1}. \tag{22}$$

To obtain an expression for the MSE, $E\{|\varepsilon(t)|^2\}$, and signal estimator channel output power,(22) is used in (14) and

$$E\{|\widehat{s(t)}|^2\} = E\{|\underline{W}_{MSE}^H(D\underline{E} + \underline{N})|^2\} \tag{23}$$

respectively to yield

$$E\{|\varepsilon(t)|^2\}\Big|_{MSE} = S + N - \underline{1}^T R_{ss} D^H [DR_{ss}D^H + R_{NN}]^{-1} DR_{ss}\underline{1} \tag{24}$$

and

$$E\{|\hat{y}(t)|^2\}\Big|_{MSE} = \underline{1}^T R_{ss} D [DR_{ss}D^H + R_{NN}]^{-1} DR_{ss}\underline{1}. \tag{25}$$

In (24) the signal and noise power have been defined as

$$S = E\{|s(t)|^2\} \tag{26}$$

and

$$N = E\{|n_o(t)|^2\}. \tag{27}$$

III. DISCUSSION OF STEADY STATE SLE PERFORMANCE

Equations (22) and (25) are the general expressions for the optimum (MSE) SLE filter weights and expected SLE output power respectively for M spectral components in a correlated noise background. The following special cases will be considered: (a) small signal to noise ratio and (b) a single spectral component. The small signal to noise ratio case will indicate that the general structure of the SLE signal estimation channel is the classical "(noise) pre-whiten and (signal) match" filter operator. The single spectral component case will illustrate that the SLE signal matched filter function is implemented by a simple Discrete Fourier Transform (DFT) filter with effective 3dB filter width, i.e. resolution, equal to $(1/L\Delta)$ Hz and center frequency equal to the frequency of the spectral component. Recall L is the number of filter taps and Δ is the temporal spacing between taps. Small signal to noise ratio: As the signal power terms on the diagonal of R_{ss} becomes small relative to the noise power terms on the diagonal of R_{NN}, the MSE weight vector $\underline{W}_{MSE}$ (20) and SLE expected output power $E\{|y(t)|^2\}$ (25) approach

$$\underline{W}_{MSE} = R_{NN}^{-1} D R_{ss} \underline{1} \tag{28}$$

and

$$E\{|y(t)|^2\}|_{MSE} = \underline{1}^T R_{ss} D^H R_{NN}^{-1} D R_{ss} \underline{1}. \tag{29}$$

First, the structure of the SLE signal estimation channel as dictated by (28) is examined. Figure 2 gives a functional block diagram of the SLE processor and indicates the important filter operations in the signal estimation channel. Observe that the matrix filter operation, R_{NN}^{-1}, on the delayed data vector $\underline{X}$ is clearly a noise prewhitening function. The scalar filter operation is equivalent to a signal matched filter. This can be seen by examining the SLE output in more detail. The output, $\hat{s}(t)$, is given by

$$\hat{s}(t) = \underline{1}^T R_{ss}^H D^H R_{NN}^{-1} \underline{X} \tag{30}$$

$$= \sum_{n,m=1}^{M} p_{nm} \underline{D}_n^H \underline{X}_w \tag{31}$$

where

$$P_{nm} = E\{\tilde{s}_n(t)\tilde{s}^*_m(t)\}\ e^{j(\omega_n-\omega_m)t} \tag{32}$$

is the correlation between the m^{th} and n^{th} spectral component, complex envelopes and $\underline{X}_w = R^{-1}_{NN}\ \underline{X}$ is referred to as the noise prewhitened data vector. The term

$$\alpha_n = \underline{D}^H_n \underline{X}_w \tag{33}$$

$$= \left[\sum_{\ell=1}^{L} x_w(t-[\ell-1]\Delta)e^{j\omega_n(\ell-1)\Delta}\right]e^{j\omega_n\delta} \tag{34}$$

$$= X_w\left(\frac{\omega_n}{2\pi}\right)e^{j\omega_n\delta} \tag{35}$$

since $X_w\left(\frac{\omega_n}{2\pi}\right)$ is recognized as the Discrete Fourier Transform (DFT) of the noise prewhitened data waveform $x_w(t)$. The DFT is evaluated at frequency ω_n. As indicated by (31), the output of the n^{th} DFT filter channel is weighted by the factor $\sum_{M=1}^{M} P_{nm}$ and summed together with the outputs of the (M-1) other correspondingly weighted DFT filter outputs. Notice that for correlated tonals, i.e. $P_{nm} \neq 0$ for $n \neq m$, coupling between DFT filter outputs occurs. This undesireable effect could lead to time varying (intermodulation) effects characterized by periodic signal suppression due to destructive combining of out of phase spectral components. This type of behavior is a distinct possibility in an adaptive realization of the SLE wherein the requirement for short adaptation time for realization of $\underline{W}_{MSE}$ (see Section IV) allows for residual correlation between complex envelope functions which, in fact, become more uncorrelated only as the correlation averaging time increases. The ideal situation, of course, is where $P_{mn} = 0$ for $m \neq n$ such that

$$\hat{s}(t) = \sum_{m=1}^{M} P_{mm}\underline{D}^H_m \underline{X}_w \tag{36}$$

and each DFT channel output is weighted proportionately to the power in the spectral line.

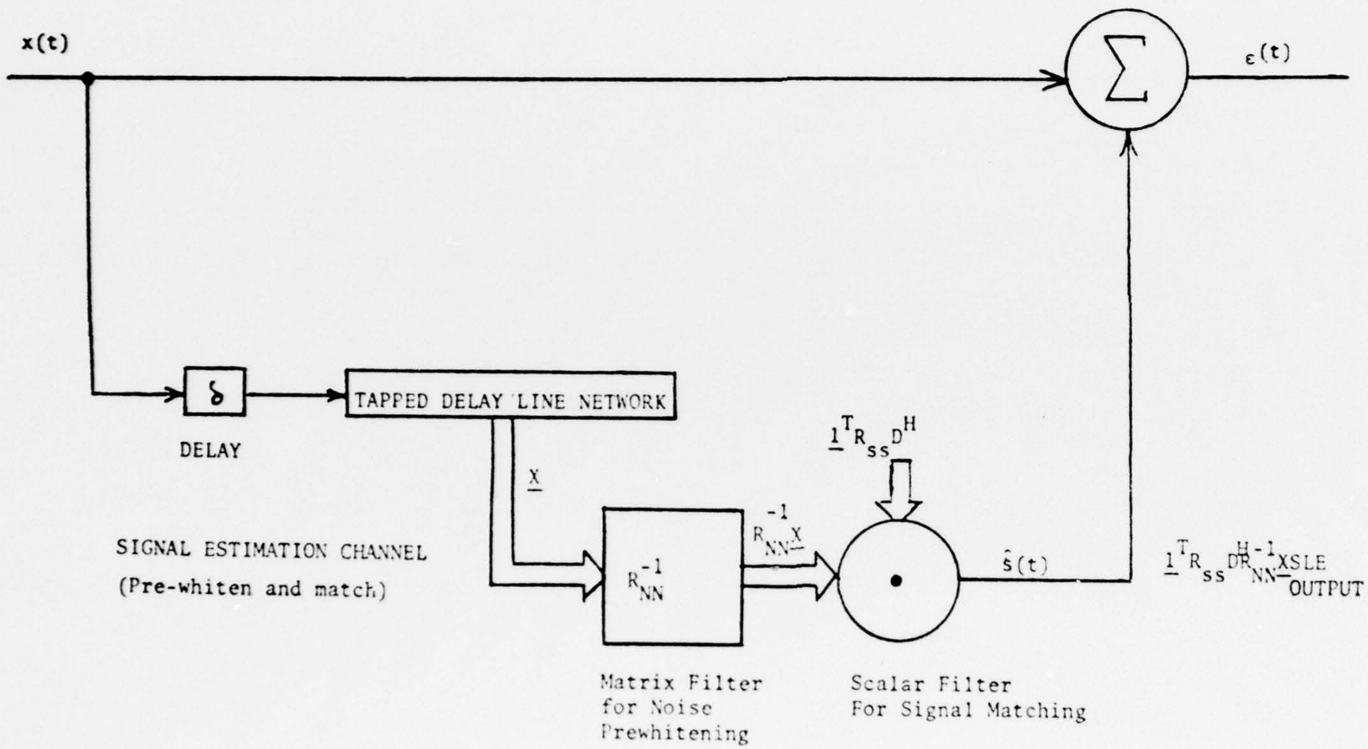


FIGURE 2. A FUNCTIONAL DIAGRAM OF THE SLE PROCESSOR (NOTE: IMPLIES A VECTOR DATA CHANNEL)

The SLE output power $E\{|y(t)|^2\}$ for the small signal to noise ratio case with uncorrelated spectral component envelopes and uncorrelated noise can be obtained from (29). For this case R_{ss} is a diagonal matrix with m^{th} element p_{mm}, the DFT matrix is such that $\underline{D}_m^H \underline{D}_n = \delta_{mn}$ (dirac delta), i.e. orthogonal carriers and $R_{NN} = NI$ where I is an L by L identity matrix. Accordingly, (29) reduces to

$$E\{|y(t)|^2\}\Big|_{MSE} = \frac{L \sum_{m=1}^{M} p_{mm}^2}{N} \tag{37}$$

It is important notice that according to (37) the only way to increase the SLE output signal to noise ratio is to increase the resolution of the adaptive DFT filters, i.e. by increasing the length of the adaptive tapped delay line L in Figure 1.

An exact expression for SLE output power for a single spectral line in additive uncorrelated noise is obtained from (25), namely ($p_{11} = S$)

$$E\{|y(t)|^2\}\Big|_{MSE} = L\frac{S^2}{N} \frac{1}{1 + L\frac{S}{N}} . \tag{38}$$

Therefore, even for small S/N if the tapped delay line length L is made sufficiently large the spectral line can be "enhanced" to the extent allowed by the resolution of the L-point DFT. The MSE weight vector $\underline{W}_{MSE}$ for the case of a single spectral line in uncorrelated noise is obtained from (20) as

$$\underline{W}_{MSE} = \frac{S}{N} \frac{1}{1 + L\frac{S}{N}} \underline{D} . \tag{39}$$

By noting that $E\{|y(t)|^2\}$ is of the form

$$E\{|y(t)|^2\} = \beta^2 L^2 S + \beta^2 LN, \tag{40}$$

the SLE output signal to noise ratio is given by

$$SNR_{out} = L\frac{S}{N} \tag{41}$$

$$= L(SNR_{in}) . \tag{42}$$

Thus, for example, a single spectral line with SNR_{in} = -20dB can be enhanced to SNR_{out} = 0 dB with an L=100 tap delay line.

A qualitative example of precisely how the SLE would function for a hypothetical spectral signal and broadband noise is informative. Figures 3(a), (b) and (c) give the power spectrum of the SLE input, the noise prewhitener output and the matched filter output. A signal consisting of two spectral lines, plus "colored" broadband noise is considered in Figure 3(a). The colored noise is characterized by a sloping spectrum which, as a result of the prewhitening matrix filter operation is "magically" transformed into a flat noise power spectrum of level N_o (Figure 3(b)). The DFT matched filters then center at the line frequencies f_1 and f_2 and exhibit filter characteristics determined by a 3dB main lobe of $1/L\Delta$ (Hz) and 13.6 dB down first sidelobes, i.e. a uniformly weighted DFT. Notice that on a per Hertz basis at f_1 and f_2 the SNR remains at 2dB and 4dB respectively. However, the broadband SNR theoretically continues to increase as L increases.

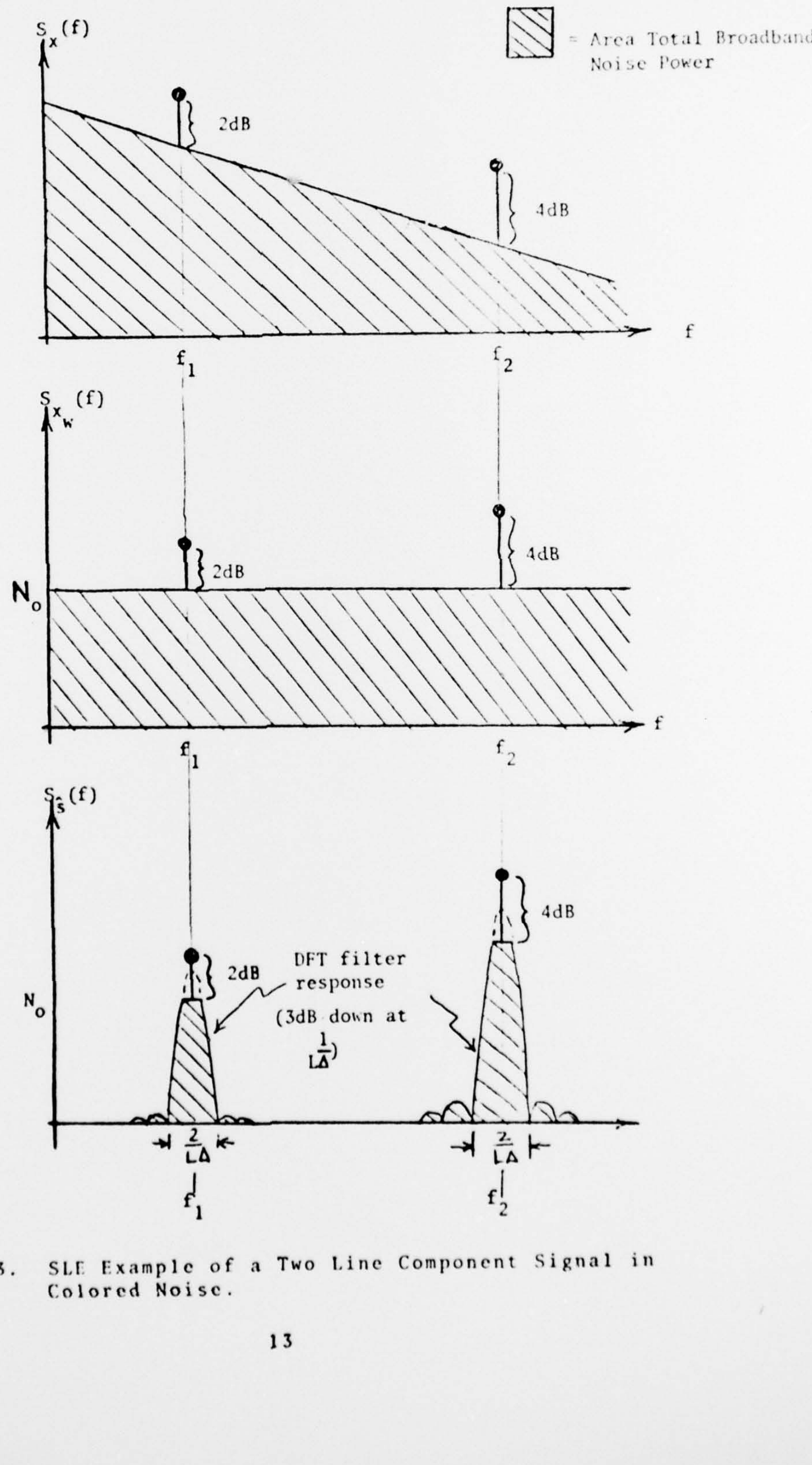


Figure 3 (a)
SLE Input
Power Spectrum

Figure 3 (b)
SLE Noise
Prewhitener Output
Power Spectrum
(Idealized)

Figure 3 (c)
SLE Matched
Filter Output
Power Spectrum
(Idealized)

FIGURE 3. SLE Example of a Two Line Component Signal in Colored Noise.

IV. THE SLE ADAPTIVE REALIZATION

An adaptive least mean square (LMS) realization of the SLE utilizing the method of steepest (gradient) descent attempts minimize the error output power at time t_i with respect to the weight vector $\underline{W}|t=t_i = \underline{W}(i)$. Therefore,

$$|\varepsilon(t_i)|^2 = \left[x(t_i) - \underline{W}^H(i)\underline{X}(i)\right]\left[x(t_i)^* - \underline{X}^H(i)\underline{W}(i)\right] \tag{43}$$

$$= |x(t_i)|^2 - 2\mathrm{Re}\{\underline{W}^H(i)\underline{X}(i)x(t_i^*)\} + |\underline{W}^H(i)\underline{X}(i)|^2. \tag{44}$$

The gradient of (43) with respect to $\underline{W}(i)$ is

$$\frac{\partial|\varepsilon(t_i)|^2}{\partial\underline{W}(i)} = -2\underline{X}(i)x(t_i)^* + 2\underline{X}(i)\underline{X}^H(i)\underline{W}(i) \tag{45}$$

$$= -2\underline{X}(i)\varepsilon^*(t_i) \tag{46}$$

which determines the steepest descent tapped delay line weight vector control algorithm

$$W(i+1) = \underline{W}(i) - 2\mu\underline{X}(i)\varepsilon^*(t_i). \tag{47}$$

In (47), the constant μ determines the rate of convergence to a steady state estimator and the stability (adaptation noise) of the final estimate of $\underline{W}_{MSE}$. The expected value of (47) after K iterations of (47) is

$$E\{\underline{W}(K)\} = \underline{M}(K)$$

$$= \left[I + 2\mu\,(DR_{ss}D^H + R_{NN})\right]^K \underline{M}(0) - \tag{48}$$

$$-2\mu\sum_{k=0}^{K-1}\left[I + 2\mu(DR_{ss}D^H + R_{NN})\right]^k DR_{ss}\underline{1} \tag{49}$$

which in the limit $K\to\infty$ can be shown to converge to (20), i.e. if μ relative to the maximum eigenvalue λ_1, of $DR_{ss}D^H + R_{NN}$ satisfies the bound

$$-\frac{1}{\lambda_1} < \mu < 0. \qquad (50)$$

For uncorrelated spectral envelopes, uncorrelated noise and resolvable spectral lines ($\underline{D}_m^H \underline{D}_n = \delta_{mn}$) the rate of enhancement for the m^{th} spectral component is controlled by the time constant

$$\tau_m = \frac{1}{1 + 2\mu(P_{mm} + N)} \cdot \qquad (51)$$

The material in this section follows previous work (for example [2]) exactly and the reader is referred to this work for additional results on convergence properties and steady state misadjustment.

V. SUMMARY

The Spectral Line Enhancer is a device for adaptive broadband noise prewhitening and spectral line extraction. It has been shown that because the SLE is essentially an adaptive discrete Fourier Transform Filter (DFT) bank (one DFT channel per spectral line) its ultimate steady state SNR performance is no better than that determined by the resolution of the DFT. Perhaps the most important aspect of the SLE is that it performs a background noise normalization function by virtue of its prewhitening capability.

REFERENCES

1. B. Widrow, R. Hearn and J. McCool, Naval Undersea Center, San Diego, Calif., Nov. 1972.

2. B. Widrow et al, "Adaptive Antenna Systems," Proceedings of the IEEE, Vol. 55, Dec. 1969, pp. 2143 - 2159.

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1. The enclosed document (1) is forwarded for your information and retention.

2. This document entitled "A Steady State Analysis of the Adaptive Spectral Line Enhancer (SLE)", was written as a follow-up to a discussion between John Neely (NAVSHIPS PMS 302-4) and the author.

3. The intent of the document is to provide PMS 302-4 with a more substantial theoretical background of the SLE hardware which is under development at NUC.

F. J. Kingsbury

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